ABSTRACT
This paper formalizes the semantics of trust and studies the transitivity of trust. On the Web, people and software agents have to interact with “strangers”. This makes trust a crucial factor on the Web. Basically trust is established in interaction between two entities and any one entity only has a finite number of direct trust relationships. However, activities on the Web require entities to interact with other unfamiliar or unknown entities. As a promising remedy to this problem, social networks-based trust, in which A trusts B, B trusts C, so A indirectly trusts C, is receiving considerable attention. A necessary condition for trust propagation in social networks is that trust needs to be transitive. However, is trust transitive? What types of trust are transitive and why? There are no theories and models found so far to answer these questions in a formal manner. Most models either directly assume trust transitive or do not give a formal discussion of why trust is transitive. To fill this gap, this paper constructs a logical theory of trust in the form of ontology that gives formal and explicit specification for the semantics of trust. Based on this formal semantics, two types of trust — trust in belief and trust in performance are identified, the transitivity of trust in belief is formally proved, and the conditions for trust propagation are derived. These results give theoretical evidence to support making trust judgment using social networks on the Web.

Categories and Subject Descriptors
J.1 [Computer Applications]: Administrative Data Processing—Business; I.2.11 [Artificial Intelligence]: Distributed Artificial Intelligence; H.4 [Information Systems]: Miscellaneous

General Terms
Theory

Keywords

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1. INTRODUCTION
The Web has been becoming an open dynamic and decentralized information/knowledge repository, a distributed computing platform and a global electronic market. In such a cyberspace, people, organizations and software agents have to interact with “strangers” and have to be confronted with various information with unfamiliar sources. This makes trust arise as a crucial factor on the web.

In general, trust is “a basic fact of social life” [21]. For example, we trust the data that we use for a decision to be reliable; when we cross an intersection of streets, we trust the cars in other directions to follow the traffic signals. Trust exists so widely in our social life that we may even not fully be aware of using it.

Trust is important to us because it is one of the foundations for people’s decisions. Generally, rational decisions made in the real world are based on the mixture of bounded rational calculation and trust. According to Simon[31], a decision making process in the real world is limited by “bounded rationality” i.e. the “rational choice that takes into account the cognitive limitations of the decision maker - limitations of both knowledge and computational capacity” . In a real decision situation, since we only have limited information/knowledge and computing capacity as well as limited time available for decision analysis, a rational decision has to be partly based on bounded rational calculation and partly based on trust. As Luhmann revealed, trust functions as “reduction of complexity” in our social life [21].

Return to the context of trust on the Web. The interest in addressing the issue of trust on the web has appeared under the umbrella of the “Web of Trust”. An important issue is how to transplant the trust mechanism in our real world into the cyberspace. To this end, many researchers attempt to construct formal models of trust and to develop tools that support people making trust judgments on the web. Basically trust is established in interaction between two entities. Many models proposed focus on how to calculate and revise trust degrees in interaction between two entities. However, one entity only has a finite number of direct trust relationships, which cannot meet the needs of various interaction with unknown or unfamiliar sources on the web. As a promising remedy to this problem, social
network-based trust, in which A trusts B, B trusts C, thus A indirectly trusts C under certain conditions, is receiving considerable attention. A necessary condition of trust propagation in social networks is that trust need to be transitive. In other words, without transitivity, trust cannot propagate in networks. However, is trust transitive? What types of trust are transitive and why? No theories and models found so far answer these questions in a formal manner. Most models mainly focus on how to calculate trust degrees when trust propagates in social networks, and they either directly assume trust transitive or do not give a formal discussion of why trust is transitive. To fill this gap, this paper aims to construct a logical theory for trust in the form of ontology, which has formal and explicit specification for the semantics of trust. From this formal semantics, two types of trust — trust in belief and trust in performance — are identified, the transitivity of trust in belief is formally proved, and the conditions for trust propagation are derived. These results provide theoretical evidence for trust propagation in social networks. The proposed trust model can be used for trust judgements using social networks on the web.

Our specific interest in trust comes from Knowledge Provenance (hereafter, referred to as KP). KP is proposed in [8, 9] to create an approach to determining the validity of web information by means of modeling and maintaining information sources, information dependencies, as well as trust structures. Three KP models, static KP [8], dynamic KP [15] and uncertainty oriented KP [16], have been studied. These models assume that trust relationships between information consumers and information creators have already been calculated, and we mainly focused on modeling and maintaining information sources and information dependencies. This paper focuses on modeling trust structures.

The contents of this paper are organized as follows: following the discussion of related research in Section 2, our view of trust is introduced in Section 3; our motivating scenarios are given in Section 4; the informal competency questions to be answered by the trust ontology under development are defined in Section 5; the methodology and terminology to be used are introduced in Section 6; the axioms and theorems in the trust ontology are developed in Section 7; an application example is given to demonstrate the potential uses of the trust ontology in Section 8; finally in Section 9, we give a conclusion and discussion on further research.

2. RELATED RESEARCH

Trust has been receiving considerable attention from many directions. This section examines the related research in two aspects: conceptualization and formalization.

2.1 Trust Conceptualization

Trust is a complex social phenomenon. The concepts developed in social sciences provide an important foundation for trust formalization.

A large body of research has contributed to the conceptualization of trust (refer to [20], [3], [10]). Rotter [29] defined trust as “expectancy”. Mayer et al [23] defined trust as “the willingness to be vulnerable”. A typical definition is “trust is a psychological state comprising the intention to accept vulnerability based upon positive expectation of the intentions or behavior of another” [30]. Deutsch [6] studied trust in cooperation with game theory, and he built up an important framework for formalizing interpersonal trust from the view of decision. Luhrmann [21] revealed “system trust” — the trust placed in the function of a system in society. Zucker [33] examined the evolution of the trust mechanism in American economic system and identified three modes of “trust production” (trust building): (1) “process-based”, in which trust is built on past history of interaction; (2) “characteristic-based”, in which trust is dependent on “social similarity”, such as culture, age and gender; (3) “institutional-based”, in which trust is built on “formal social structure” comprising of “membership in a subculture” and “intermediary mechanism”, such as regulation, legislation, functions of governments and banks.

Most of the study of trust in social sciences focuses on inter-individual trust, but much attention has shifted to “system trust” — a different mechanism of trust emerging at the turn of last century to meet people’s increasing demands of more and more interaction with strangers in industrial society. This situation is very similar to the trust problem we are facing today in the cyberspace.

2.2 Trust Formalization

In computer science, trust was initially concerned by the security community. For the purposes of secure web access control, Blaze et al [2] first proposed “decentralized trust management” to separate trust management from applications, and developed the PolicyMaker system. Khare and Rifkin [19] proposed basic principles of trust management.

Trust is also a concern of the distributed AI community [26]. Marsh [22] constructed a formal model of trust to describe the relations among the major concepts of trust. His study is among the first to formalize trust. However, on one hand, his work solely focuses on inter-individual trust; on the other hand, trust is represented with simple arithmetic formulas, and the logical relations in trust were not explicitly described. Demolombe [5] constructed a modal logic system of trust. This model defines the formal semantics of trust as belief based on properties of sincerity, credibility, cooperativity and vigilance. However, an important element of trust — context is not considered in the semantics of trust, and the transitivity of trust is not studied either. Gans et al [11] addressed distrust in trust modeling. Falcone & Castelfranchi [7] developed a formal trust model based on BDI agent architecture. One of the features of this model is the representation of the aspects of competence and willingness of trust. Perhaps because trust problem in DAI arises in the interaction among agents, all the work discussed above focus on inter-individual trust.

In recent years, trust models based on social networks are receiving considerable attention. Particularly, the trend is powered by “web of trust” which is identified as the top layer of the semantic web. The concept of “web of trust” perhaps is first developed in PGP as a trust model used for public key validation by using social networks. However, “trust” in PGP is specifically on public key validation. FOAF project (http://foaf-project.org/) attempts to create social networks on the web by facilitating people to describe acquaintance relationships in machine-readable web pages. Although ac-
quaintance relationships are not trust relationships, FOAF is a good starting point. Recently, many models of trust on the web using social networks have emerged. For examples, Yu and Singh [32] constructed a model of reputation (trust) updating and propagation using the testimonies from the trustee’s neighbors; Golbeck et al [32] extended the acquaintance relationships in FOAF model by introducing levels of trust and applied the model for filtering emails; Richardson et al [28] proposed an algebra representation of trust propagation and applied it in bibliography recommendation; Guha and Kumar [14] constructed a trust propagation model considering distrust; Abdul-Rahman and Hailes [1] proposed a trust model including “direct trust” and “recommender trust”, in which trust propagates in social networks; Josang et al [18] argued that trust is not always transitive and “referral trust” (i.e. “recommender trust”) is transitive. However, they did not reveal why recommendation is transitive in a formal manner. The common perspective of all these models is trust propagation in social networks. However, is trust transitive? What types of trust are transitive and why? No theories and models found have answered these questions in formal manner. The models found either directly assume trust transitive or do not give formal discussion why trust is transitive, due to no formal representation of the semantics of trust.

In order to formally study the transitivity of trust, the formal representation of the semantics of trust is necessary. However, no formal models found meet the needs for such a analysis. To fill this gap, we aim to identify the semantics of trust, to develop a logical model of trust to reveal the logical relations among the constructs of trust, and from this formal semantics of trust, to prove the transitivity of trust.

3. WHAT IS TRUST?

Synthesizing the concepts of trust in trust related literature, we have the following view of trust.

Trust is the psychological state comprising (1) expectancy: the trustee expects a specific behavior of the trustee such as providing valid information or effectively performing cooperative actions; (2) belief: the trustee believes that expectancy is true, based on evidence of the trustee’s competence and goodwill; (3) willingness to be vulnerable: the trustee is willing to be vulnerable to that belief in a specific context where the information is used or the actions are applied.

In trust, there are two roles involved: trustor and trustee. Furthermore, there are three aspects in the implications of trust. First, when it is said that entity A trusts B, people must ask a question “on what thing does A trust B?” This leads to the first implication of trust: expectancy, i.e. the trustee expects that trustee behaves in a specific manner within a specific context. The expected behavior can be: valid information or cooperative actions. Secondly, when trusting, the trustee must believe that the trustee behaves as expected, according to the competence and goodwill of the trustee. This is the most recognized aspect of the meaning of “trust”. Thirdly, the trustee not only believes the trustee will behave as expected but also is willing to be vulnerable for that belief in a specific situation, i.e. the trustee is willing to assume the risk that the trustee may not behave as expected.

As many researchers realized, trust is context-specific, for example, a person may trust her or his financial advisor about investment analysis but doesn’t trust the advisor in health-care. There are two types of contexts related to trust. The first is the context where the expected valid information is created, and the second is the context in which the trusted information (or action) will be applied by trustor or the situation where the trustor is confronted with the trust judgment problem. These two contexts may not be the same. For example, a financial expert (the trustee) created a piece of information in a context of giving financial investment seminar, and in another context of buying stocks, an investor (the trustor) attempts to use this information and needs to judge the validity of it.

In summary, the meaning of trust we adopt can be expressed as follows:

\[ Trust = \text{Expectancy} + \text{Belief in expectancy} + \text{Willingness to be vulnerable for that belief}. \]

4. MOTIVATING SCENARIOS

In order to construct a trust ontology, we start from motivating scenarios.

Consider the following use cases of the electronic business of a gift company (denoted as F). F originally is a floral company. Now its online business includes both bouquets and other gift products. The web services technology makes F able to extend its online store to a virtual omni-gift store that sells not only the products in its catalogues but also any other types of gift products available via web services.

Case 1: Trust in Business Partnerships

James, an old customer of F, wants to order from F’s online store a gift to a friend, a special high quality fine china tea set made in Jingdezhen of China. Assume this product is beyond F’s catalogues. In F’s online store, a sales agent, a software robot representing F, first finds from web services registries (e.g. UDDI registries) a list of service providers who sell the requested products; second, from this list, as a general business rule, the agent needs to check out service providers whose products are trusted by F to have high quality; then, the agent presents to James a selected list of products provided by the trusted service providers; after James chooses a satisfied product, the agent orders the product for him. Here, we ignore the detail of web service process and only focus on how the agent makes trust judgments.

A service provider which F found in a web services registry is J, a porcelain company in Jingdezhen city. In order to maintain F’s business reputation, before F recommends J’s fine china tea sets to customers, F needs to make sure J’s products with high quality. To make this judgment, the sales agent searches J in F’s business relationship management system. Unfortunately, J is not in the system, that is to say, F does not know J at the time. However, F has a long term business partner P. P frequently provides for F porcelain products such as vases. Therefore, F trusts P’s judgments on porcelain product quality. Furthermore, P has a major trusted porcelain product supplier S in New York, so sim-
ilarly P trusts S’s judgment on porcelain product quality. Finally J is one of S’s trusted suppliers. In this latter relationship, S trusts the product quality of J. From this chain of trust, the sales agent of F infers that the porcelain teaset s of J have high quality.

Now we make further analysis on the roles and relationships demonstrated in this case.

James, a customer of F who trusts F’s services, wants to buy from F a special high quality fine china tea set. This request activates web service discovery, trust judgments, and trading.

F, a gift company, trusts P on P’s judgement on porcelain product quality because P is a long term professional business partner of F. In the terms of trust, the expectancy of F to P is that P’s judgement on porcelain product quality is correct, and F feel comfortable to believe what P believes in porcelain product quality.

P, a porcelain product company, trusts S’s judgment on porcelain product quality similarly as F does.

S, a porcelain product supplier, trusts J on the quality of the porcelain products J produces. In the terms of trust, S expects that the porcelain products of J have high quality; based on J’s performance in the past, S feels like to believe this expectancy.

J, a porcelain manufacturer at Jingdezhen, is unknown to F.

The chain of trust can be summarized as follows: (1) S trusts that the porcelain products of J have high quality; (2) P trusts S’s judgement on porcelain product quality, so P also trusts that the porcelain products of J have high quality; (3) similarly, F trusts P’s judgement on porcelain product quality. This makes F to trust that the porcelain products of J have high quality.

Case 2: Trust in System
Consider another situation in which F does not know J, and F also does not find any useful relationship in its business relationship management system to connect to J. However, J shows that it has ISO 9000 quality certification. Assume that F trusts ISO 9000 system thus F trusts J’s products meeting ISO 9000 quality standards. This makes F to trust that the porcelain products of J have high quality.

Case 3: Trust in Business Partnerships (2)
Consider the business relationships in case 1. Assume that S trusts P that P is able to and will pay for all its purchases so allows P to pay quarterly rather than to pay in every transaction, and P trusts F in the same way. Now, consider the following situation. For the first time, F orders a considerable amount of porcelain vases directly from S. Does S trusts F on later payment as trusts P? The common knowledge tell us that the answer is “no”. This is because the facts that S trusts P to pay later and P trusts F to pay later does not necessarily imply S trusts F to pay later. In other words, this trust is not transitive.

Findings:
From these cases, a number of important facts and concepts can be revealed.

Trust Factor in Web Services Based B2B
First, in the e-business based on web services technology, a web services user (here, F) accepts only trusted service providers retrieved from web services registries. The trust issues may arise in many aspects of e-business including service quality, business credibility, as well as business information security and privacy.

Types of Trust
There are different types of trust.

In case 1, the trust which F places on P is the trust in what P believes. The trust P places on S is the same type. From these examples, we can identify a type of trust: trust in belief. The trust which P places on F in case 3 is the trust in F’s performance. This is another type of trust: trust in performance. The other examples of this type of trust include the trust which S places on P in case 3 and S places on J in case 1.

Transitivity of Trust
In case 1, S’s trust in J propagates to P and then to F because F trusts in P’s belief and P trusts in S’s belief on product quality. This fact leads us to a general fact: trust in belief is transitive. Because of this property, trust can propagate in business relationship formed social networks.

However, trust is not transitive in general. Case 3 shows that trust in performance is not transitive.

Sources of Trust
Basically, trust comes from the experience of interaction between two parties. For example, F trusts in P’s belief on porcelain product quality because P is F’s long term business partner and F gradually knows that P is very professional on porcelain business from the business interaction between them. The experience of interaction is the essential source of trust. The trust from interaction experience is called inter-individual trust, and also called direct trust.

Interestingly, from case 1, we have observed the fact that trust may propagate in social networks. Because of trust propagation, F trusts in J’s products. This leads to a new type of trust, whose source is from trust propagation in social networks. In this paper, this type of trust is called relational trust or social networks based trust.

Finally, in case 3, F trusts in J’s products because F trusts in ISO 9000 quality management system and J complies with the system. This is an example of system trust that we discussed in section 2.

5. INFORMAL COMPETENCY QUESTIONS
In order to give a formal and explicit specification for trust to make it able to be used in trust judgments using social networks on the web, we develop a logical theory of trust in the form of ontology.
Following the ontology development methodology of Gruninger & Fox [13], we specify trust ontology in 4 steps: (i) provide motivating scenarios; (ii) define informal competency questions for which the ontology must be able to derive answers; (iii) define the terminology; (iv) formalize competency questions, and develop the axioms (i.e. semantics) of the ontology to answer these questions. We already discussed motivating scenarios in the earlier section. This section presents informal competency questions.

In the context of making trust judgments in social networks, the trust ontology under development needs to answer the following competency questions:

Q1: In a specific context, can an entity (trustor) trusts another entity (trustee) regarding what the trustee performs, particularly, the validity of the information created by trustee?

Q2: In a specific context, can an entity (trustor) trusts another entity (trustee) regarding what the trustee believes?

Q3: When an entity has trust in another entity’s belief in a given context, and the second entity has trust in a third entity’s performance (or belief) in another context, can the first entity have trust in the third entity’s performance (or belief)? If so, in what context?

Q4: When an entity has trust in the performance of an organization (or a group whose members have a common set of characteristics related to the trust) in a specific context, can this entity have trust in the performance of the members of this organization or group?

Q5: When an entity has trust in another entity’s performance (or belief) in a specific context, given the information created by the second entity within the context, can the first entity believe this information?

After we define the terminology of the ontology under development, these informal competency questions will be formally represented, then axioms will be developed to answer these competency questions.

6. METHODOLOGY AND TERMINOLOGY

6.1 Methodology

In order to define our trust ontology, we formalize trust by using the situation calculus. The situation calculus is a many-sorted logic language. We focus on the logic of trust and to make our model easier to be used, we will represent contexts with fluents in the situation calculus.

The logical model of trust under development mainly focuses on the logical relations among trust related fluents. These relations are called state constraints in the language of situation calculus, and methods have been developed to solve the so called “ramification problem” [25]. In order to focus on the logic of trust and to make the theory easy to be understood, this paper remains the theory in the form of state constraints. Although actions and their preconditions as well as successor state axioms are major contents of situation calculus, this paper will not discuss these contents for the same reason above, and we will discuss them in another paper.

In this paper, we only consider the case of certainty, thus believing will certainly lead to willing to be vulnerable. For this reason, we will not explicitly include “willing to be vulnerable” in our formalization. In the cases of uncertainty in which the degree of belief is considered, willingness to be vulnerable for that belief is corresponding to a decision to trust or not (refer to [17]).

Finally, we follow the convention that all unbound variables are universally quantified in the largest scope.

6.2 Terminology

We define the relational fluents, functional fluents and predicates to be used in our trust ontology in tables 1, 2 and 3 respectively.

7. AXIOMS AND THEOREMS

7.1 Formal Semantics of Trust

As revealed in the motivating scenarios, two types of trust are identified in accordance with the types of expectancies: trust in performance, and trust in belief. We formally define them one by one as follows.
### Table 1: Relational Fluents

<table>
<thead>
<tr>
<th>Fluent</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>believe(d, x)</td>
<td>( E \times F )</td>
</tr>
<tr>
<td>trust_p(d, e, x, k)</td>
<td>( E \times E \times F \times F )</td>
</tr>
<tr>
<td>trust_b(d, e, x, k)</td>
<td>( E \times E \times F \times F )</td>
</tr>
<tr>
<td>has_p_tr(d, e, x, k)</td>
<td>( E \times E \times F \times F )</td>
</tr>
<tr>
<td>has_b_tr(d, e, x, k)</td>
<td>( E \times E \times F \times F )</td>
</tr>
<tr>
<td>made(x, d, k)</td>
<td>( F \times E \times F )</td>
</tr>
<tr>
<td>memberOf(e, o)</td>
<td>( E \times E )</td>
</tr>
</tbody>
</table>

### Table 2: Functional Fluents

<table>
<thead>
<tr>
<th>Fluent</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>info(x)</td>
<td>( D \rightarrow F )</td>
</tr>
<tr>
<td>perform(e, x)</td>
<td>( E \times A \rightarrow F )</td>
</tr>
</tbody>
</table>

### Table 3: Predicates

<table>
<thead>
<tr>
<th>Predicate</th>
<th>Definition/Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>holds(p, s)</td>
<td>( F \times S ) as defined in situation calculus, fluent p holds in situation s.</td>
</tr>
<tr>
<td>Poss(a, s)</td>
<td>( A \times S ) It is possible to perform action a in situation s.</td>
</tr>
<tr>
<td>entail(q, k)</td>
<td>( F \times F ) Context q entails context k.</td>
</tr>
</tbody>
</table>

7.1.1 Trust in performance

Trust in performance is the trust in what trustee performs such as the information created or the actions performed.

Trust in performance has the following basic semantics: the trustor believes in a piece of information created by the trustee in a context within the trustor’s context of trust; or, the trustor believes in the performance of an action committed by the trustee in a context within the trustor’s context of trust. This semantics can be formally defined as the following axiom:

**Axiom 1** (formal semantics of trust in performance):

\[
\text{holds(trust}_p(d, e, x, k), s) \equiv \forall q. (\text{holds}(\text{made}(x, e, q, s) \land \text{entail}(q, k)) \\
\supset \text{holds}(\text{believe}(d, k \supset x), s))
\]

(4)

where predicate entail(q, k) is defined as follows.

**Definition:**

\[
\text{entail}(q, k) \equiv \forall s, (\text{holds}(q, s) \supset \text{holds}(k, s))
\]

(5)

In this axiom (4), the expected thing (called expectancy) is fluent x; believe(d, k \supset x) represents that d believes x to be true when context k is true. In other words, d believes x in context k.

To express such a trust relationship in practice, variable d will be bound to a trustor; variable e will be bound to a trustee; variable x will be bound to a fluent representing the information created by trustee; variable k will be bound to a fluent representing the context of trust.

**Example 1.** Ben, a customer of Amazon, trusts Amazon regarding the message that his order status is “Shipped” in the context of an online trading. This trust relationship, which holds in an situation after Ben made his order, can be represented as follows:

\[
\text{holds(trust}_p(Ben, Amazon, \\
\text{order}_st(#102579, Shipped), \\
\text{order}_from(#102579, Amazon) \\
\land \text{order}_st(#102579, Shipped), S_0)
\]

According to the axiom 1, this means that for any context q, if Amazon creates information

\[
\text{order}_st(#102579, Shipped)
\]

in q, and q entails context

\[
\text{order}_st(#102579, Shipped) \\
\land \text{order}_from(#102579, Amazon),
\]

then Ben believes this information in this context of the trust. The formal representation of this meaning is as fol-
trust relationship can be represented as follows:

\[
\text{\textit{holds}}(\text{\textit{trust\_p}}(\text{\textit{Ben}}, \text{\textit{Amazon}}, \\
\text{\textit{perform}}(\text{\textit{Amazon}}, \text{\textit{return}}(#102579)), \\
\text{\textit{refund\_asked}}(#102579) \\
\wedge \text{\textit{meets\_p}}(#102579)), \\
s))
\]

The meaning of this trust relationship is similar to example 1 except the expectancy is

\[
\text{\textit{perform}}(\text{\textit{Amazon}}, \text{\textit{return}}(#102579)).
\]

From its basic semantics, trust in performance can be extended to a **stronger semantics**: the trustor believes everything (rather than one specific thing) made by the trustee in a context within the trustor’s context of trust. This stronger semantics can be formally represented as the following theorem.

**Theorem 1.**

\[
(\forall x, \text{\textit{holds}}(\text{\textit{trust\_p}}(d, e, x, k), s) \equiv \\
(\forall y, q)(\text{\textit{holds}}(\text{\textit{made}}(y, e, q), s) \wedge \text{\textit{entail}}(q, k\theta) \\
\supset \text{\textit{holds}}(\text{\textit{believe}}(d, k\theta \supset y), s))
\] (7)

Here,

\[
\theta = \{x/y\}
\] (8)

is a substitution operator that replaces variable \(x\) with term \(y\) in the operand. In this theorem, the operand is the fluent which \(k\) is bound to.

(The proof of this theorem is omitted here.)

In this theorem, sentence

\[
\forall x, \text{\textit{holds}}(\text{\textit{trust\_p}}(d, e, x, k), s)
\] (9)

represents the stronger or general trust relationship: trustor \(d\) trusts trustee \(e\) regarding any information \(x\) trustee creates in the context of trust \(k\).

To express this stronger trust relationship, \(x\) needs to remain as variable to represent the expectancy of the trust; the context of trust, which variable \(k\) will be bound to, may addresses the expectancy of trust (i.e. \(x\)).

**Example 3.** Sentence

\[
\forall x, \text{\textit{trust\_p}}(\text{\textit{Bill}}, \text{\textit{Greenspan}}, x, \\
inFieldOf(x, \text{\textit{Economic\_Analysis}}))
\]

represents that Bill trusts Greenspan regarding what Greenspan states in the field of economic analysis.

This trust relationship means: for every statement (denoted as \(y\)) made by Greenspan in a context \(q\) within the context of trust (i.e. in field of economic analysis), Bill will believe this statement in that context.

The thing (denoted as \(x\)) on which Bill trusts Greenspan is the same thing (denoted as \(y\)) which Greenspan performs.

Corollary (trust in performing actions):

\[
\text{\textit{holds}}(\text{\textit{trust\_p}}(d, e, \text{\textit{perform}}(e, a), k), s) \equiv \\
\forall q, (\text{\textit{holds}}(\text{\textit{made}}(\text{\textit{perform}}(e, a), e, q), s) \wedge \text{\textit{entail}}(q, k) \\
\supset \text{\textit{holds}}(\text{\textit{believe}}(d, k \supset \text{\textit{perform}}(e, a)), s))
\] (6)

This corollary states that at any situation \(s\), trustor \(d\) trusts trustee \(e\) on performing action \(a\) in context \(k\) is logically equivalent to that if \(e\) commits to do \(a\) in context \(q\) that is within \(k\) then \(d\) believes in that \(e\) performs \(a\) in context \(k\).

**Example 2.** Ben trusts Amazon regarding getting refund for an order in the context that he wants get refund for some reasons and the order meets Amazon’s returns policy. This
Technically, in order to match these two things as one, substitution $\theta$ is employed to unify $x$ with $y$. After this substitution, the thing $x$ addressed in the context of trust $(k)$ is bound to the same thing $y$ as addressed in the context of performance $(q)$.

### 7.1.2 Trust in belief

**Trust in belief** is the trust placed on what trustee believes.

The basic semantics of trust in belief is that the trustor believes a thing believed by the trustee in a context within the trustor’s context of trust. This is formally defined in the following axiom.

**Axiom 2 (formal semantics of trust in belief):**

$$\forall q, \text{holds} (\text{believe} (e, q \supset x), s) \land \text{entail} (q, k) \supset \text{holds} (\text{believe} (d, k \supset x), s)$$ (10)

**Example 4.** In the context of our motivating example, F trusts what P believes regarding porcelain product, e.g., “TeaSet-J1106b”. This can be represented as follows.

$$\text{holds} (\text{trust}_b (F, P, \text{qual}_\text{grade} (\text{TeaSet} – J1106b, A), \text{in_topic} (\text{qual}_\text{grade} (\text{TeaSet} – J1106b, A), \text{Porc}_\text{Qual})), s),$$ (11)

where $\text{qual}_\text{grade} (\text{TeaSet} – J1106b, A)$ represents the quality grade of product TeaSet-J1106b is $A$.

Similar to trust in performance, trust in belief has the stronger semantics that the trustor believes everything believed by the trustee in a context that is within the trustor’s context of trust. The formal representation of this semantics is as follows.

**Theorem 2.**

$$\forall x (\text{holds} (\text{trust}_b (d, e, x, k), s) \equiv \forall y, q (\text{holds} (\text{believe} (e, q \supset y), s) \land \text{entail} (q, k) \supset \text{holds} (\text{believe} (d, k \supset y), s))$$ (12)

**Example 5.** In our motivating example, the trust relationship that F trusts what P believes regarding porcelain product quality can be represented as follows.

$$\forall x, \text{holds} (\text{trust}_b (F, P, x, \text{in_topic} (x, \text{Porc}_\text{Qual})), s)$$ (13)

### 7.2 Sources of Trust

In the last subsection, we discussed the semantics of trust or what trust means. This subsection discusses where trust comes from.

Trust comes from three types of sources: (1) direct trust from the experience of interaction between trustor and trustee, called inter-individual trust; (2) trust derived through trust propagation in social networks, called relational trust; (3) trust via the trust placed on the stable or predictable functions / behaviors of a system, called system trust, typically including institutional based trust, professional membership based trust, as well as characteristics based trust. The above (2) and (3) are also called indirect trust.

Inter-individual trust relationships are represented with fluent: $\forall x, \text{has}_p \text{tr} (d, e, x, k)$, which represents trustor $d$ has trust in performance type of inter-individual trust relationship with trustee $e$ in context $k$, and $\forall x, \text{has}_b \text{tr} (d, e, x, k)$, which represents trustor $d$ has trust in belief type of inter-individual trust relationship with trustee $e$ in context $k$.

Inter-individual trust also have the same semantics as the general concept of trust. Therefore, the general semantics of trust is the necessary condition of inter-individual trust as described in the following axiom.

**Axiom 3.**

$$\text{holds} (\text{has}_p \text{tr} (d, e, x, k), s) \supset \forall y, q \text{holds} (\text{made} (e, q \supset y), s) \land \text{entail} (q, k) \supset \text{holds} (\text{believe} (d, k \supset y), s)$$ (14)

$$\text{holds} (\text{has}_b \text{tr} (d, e, x, k), s) \supset \forall y, q \text{holds} (\text{believe} (e, q \supset y), s) \land \text{entail} (q, k) \supset \text{holds} (\text{believe} (d, k \supset y), s)$$ (15)

The sufficient condition for inter-individual trust depends on how trust is built up in the process of interaction among entities. Many researchers have studied this problem (refer to Section 2). In this paper, we will not discuss how inter-individual trust relationships are built up; instead we only assume that entities in social networks have developed their inter-individual trust relationships, from which the indirect trust relationships are derived.

The following theorem states that a trust relationship holds if the inter-individual trust relationship holds.

**Theorem 3.**

$$\forall x, \text{holds} (\text{has}_p \text{tr} (d, e, x, k), s) \supset (\forall x, \text{holds} (\text{trust}_p (d, e, x, k), s))$$ (16)

$$\forall x, \text{holds} (\text{has}_b \text{tr} (d, e, x, k), s) \supset (\forall x, \text{holds} (\text{trust}_b (d, e, x, k), s))$$ (17)

This theorem needs to be used for trust inference.

### 7.3 System Trust

As discussed earlier, system trust is the trust placed on the stable or predictable functions or behaviors of a system. Case 2 in the motivating scenarios is an example of using system trust. The typical forms of system trust include professional membership-based trust, characteristics-based trust, institutional based trust or regulation-based trust. System trust is a wide topic which cannot be covered in a short
Theorem 4.
Theorem 4 and 5 can be proved directly from axioms about theorems 4, 5 and 6 provide rules for such type of reasoning.

System trust makes trust in performance able to be transferred from an organization or a group to its members. This axiom is illustrated in figure 1(c). This is one of the conditions for trust propagation in social networks.

An example of using system trust in a trust judgment is given in the second case of section 8.

7.4 Reasoning with Trust Relationships
The above discussed axioms and theorems can be used to infer whether a thing can be believed. When an individual (trustor) is confronted by a thing (information or actions) made or believed by another individual (trustee), the trustor can infer whether or not to believe this thing by considering whether the trustee trusts the trustee. The following theorems 4, 5 and 6 provide rules for such type of reasoning. Theorems 4 and 5 can be proved directly from axioms about the semantics of trust.

Theorem 4.
\[(\forall x)(\text{holds}(\text{trust}_d(d, e, x, k), s)) \land \text{holds}(\text{made}(y, e, q), s) \land \text{entail}(q, k\theta) \supset \text{holds}(\text{believe}(d, k\theta \supset y)), s)\] (19)

In this and the following theorems, substitution \(\theta\) is the same as defined in (8).

Theorem 5.
\[(\exists z)(\text{holds}(\text{trust}_b(d, e, x, k), s)) \land \text{holds}(\text{believe}(e, q \supset y), s) \land \text{entail}(q, k\theta) \supset \text{holds}(\text{believe}(d, k\theta \supset y)), s)\] (20)

The following theorem states that if entity \(d\) believes fluent \(x\) in context \(k\) in a situation \(s\), then when \(d\) believes \(k\) in \(s\), \(d\) also believes \(x\) in \(s\).

Theorem 6.
\[\text{holds}(\text{believe}(d, k \supset x), s) \land \text{holds}(\text{believe}(d, k), s) \supset \text{holds}(\text{believe}(d, x), s)\] (21)

The proof of this theorem is omitted here, for the proof related to the representation of belief, and we do not discuss belief further in this paper.

Theorem 7 reveals that if \(d\) trust \(e\) on thing \(x\) in context \(k\), then \(d\) also trusts \(e\) on \(x\) in a stricter context (that satisfies \(k\)).

Theorem 7.
\[\text{holds}(\text{trust}_d(d, e, x, k), s) \land \text{entail}(q, k) \supset \text{holds}(\text{trust}_d(d, e, x, q), s)\] (22)

\[\text{holds}(\text{trust}_b(d, e, x, k), s) \land \text{entail}(q, k) \supset \text{holds}(\text{trust}_b(d, e, x, q), s)\] (23)

7.5 Transitivity of Trust
Trust propagation in social networks is based on the assumption that trust is transitive. However, trust is not always transitive. For example, \(A\) trusts \(B\) to access \(A\)’s resources, and \(B\) trusts \(C\) to access \(B\)’s resources, but these do not necessarily imply that \(A\) trusts \(C\) to access \(A\)’s resources. The question interesting here is what type of trust is transitive.

Our answer to the question is that trust in belief is transitive. We give the formal description of the result as the following theorem.

Theorem 8 (Transitivity of trust in belief).
(a) In any situation \(s\), if entity \(d\) trusts entity \(c\) on everything which \(c\) believes in context \(k\), and \(c\) trusts entity \(e\) on everything which \(e\) performs in context \(q\), then \(d\) trusts \(e\) on everything which \(e\) performs in the conjunction of contexts \(k\) and \(q\).

\[(\forall x)(\text{holds}(\text{trust}_b(d, c, x, k), s)) \land (\forall x)(\text{holds}(\text{trust}_c(c, e, x, q), s)) \supset (\forall x)(\text{holds}(\text{trust}_d(d, e, x, k \land q), s))\] (24)

Proof. To prove this theorem, by theorem 1 of (7), we need to prove:

\[(\forall x, p)(\text{holds}(\text{made}(x, e, p), s) \land \text{entail}(p, k \land q)) \supset \text{holds}(\text{believe}(d, k \land q \supset x), s),\] (25)

from the given premises:

\[(\forall x)(\text{holds}(\text{trust}_b(d, c, x, k), s))\] (26)

\[(\forall x)(\text{holds}(\text{trust}_c(c, e, x, q), s)).\] (27)

Note that all variables are arbitrary.

Assume

\[\text{holds}(\text{made}(x, e, p), s)\] (28)

and

\[\text{entail}(p, k \land q).\] (29)
From the definition of entail (5) and the “propositional functions” of fluents (refer to (1)), we have:

entail\((k \land q, k)\) \hspace{1cm} (30)
entail\((k \land q, q)\) \hspace{1cm} (31)
entail\((k \land q, k \land q)\) \hspace{1cm} (32)

From theorem of (22)/(23), premises (26), (27), and the above (30) and (31), we have,

\((\forall x)(\text{holds}(\text{trust}_b(c, e, x, k \land q), s))\) \hspace{1cm} (33)
\((\forall x)(\text{holds}(\text{trust}_b(d, c, x, k \land q), s))\) \hspace{1cm} (34)

From theorem of (19), assumptions (28) and (29), as well as (33), we have

\(\text{holds}(\text{believe}(c, k \land q \supset x), s))\) \hspace{1cm} (35)

So, sentence (25) is proved. Note that all variables are arbitrary although \(x\) is unified. So, the theorem is proved.

(b) In any situation \(s\), if entity \(d\) trusts entity \(c\) on everything which \(c\) believes in context \(k\), and \(c\) trusts entity \(e\) on everything which \(e\) believes in context \(q\), then \(d\) trusts \(e\) on everything which \(e\) believes in the conjunction of the contexts \(k\) and \(q\).

\((\forall x)(\text{holds}(\text{trust}_b(d, c, x, k), s)) \land (\forall x)(\text{holds}(\text{trust}_b(d, c, x, q), s)) \supset (\forall x)(\text{holds}(\text{trust}_b(d, c, x, k \land q), s))\) \hspace{1cm} (37)

This part of theorem can be proved with the same methods and proof structure as the proof of part (a), and we just need to replace \(\text{trust}_p\) with \(\text{trust}_b\). This part of proof is omitted.

This theorem shows that trust in belief is transitive. Regarding trust in performance, as revealed in case 3 of the motivating scenarios, this type of trust relationship generally is not transitive. However, trust in performance can be derived by using trust in belief (theorem 8(a)) and by using system trust (axiom 4 in section 7.3).

Theorem 8 together with axiom 4 also described three conditions and forms of trust propagation: (1) \(A\) has \(\text{trust in belief}\) relationship with \(B\), \(B\) has \(\text{trust in belief}\) relationship with \(C\), then we can derive that \(A\) has \(\text{trust in belief}\) relationship with \(C\); (2) \(A\) has \(\text{trust in belief}\) relationship with \(B\), \(B\) has \(\text{trust in performance}\) relationship with \(C\), then it can be derived that \(A\) has \(\text{trust in performance}\) relationship with \(C\); (3) \(A\) has \(\text{trust in performance}\) relationship with

Figure 1: Three forms and conditions of trust propagation in social networks

B, B is a system (organization or group) and has member C, then it can be derived that A has \(\text{trust in performance}\) relationship with C. These are illustrated in figure 1. For simplicity, we assume the context of trust is the same and omitted in the figure.

8. APPLICATION EXAMPLE

In the following, to demonstrate the potential uses of the proposed trust ontology in trust judgments using social networks on the web, we apply the trust ontology to the trust judgment problem in web services base e-business introduced in the motivating scenarios.

Case 1: the trust relationships in case 1 of the motivating scenarios is formally represented as follows.
(1) F trusts in P’s belief on porcelain product quality.

\[
\text{holds}(\text{has}_b \text{tr}(F, P, x, \text{in}_\text{topic}(x, \text{PorcQual})), s) \quad (38)
\]

Here x is a free variable, so it universally quantified in the largest scope.

(2) P trusts in S’s belief on porcelain product quality.

\[
\text{holds}(\text{has}_b \text{tr}(P, S, x, \text{in}_\text{topic}(x, \text{PorcQual})), s) \quad (39)
\]

Similar to (1), x is a free variable.

(3) S trusts in J’s performance on high quality porcelain product manufacture.

\[
\text{holds}(\text{has}_b \text{tr}(S, J, \text{perform}(J, \text{makeHGPorcln}(x)), \text{in}_\text{topic}(\text{perform}(J, \text{makeHGPorcln}(x)), \text{PorcQual}), s)) \quad (40)
\]

Here, term \( \text{makeHGPorcln}(x) \) represents J’s performance on high quality porcelain product manufacture.

Now, we use the above facts and the proposed trust ontology to answer whether F trusts in J’s performance on high quality porcelain product manufacture.

From (38) and theorem 3b (17), we have,

\[
\text{holds}(\text{trust}_b(F, P, x, \text{in}_\text{topic}(x, \text{PorcQual})), s) \quad (41)
\]

From (39) and theorem 3b (17), we have,

\[
\text{holds}(\text{trust}_b(P, S, x, \text{in}_\text{topic}(x, \text{PorcQual})), s) \quad (42)
\]

From (41), (42) and theorem 8a (24), we have,

\[
\text{holds}(\text{trust}_b(F, S, x, \text{in}_\text{topic}(x, \text{PorcQual})), s) \quad (43)
\]

From (40) and theorem 3a (16), we have,

\[
\text{holds}(\text{trust}_p(S, J, \text{makeHGPorcln}(x)), \text{in}_\text{topic}(\text{perform}(J, \text{makeHGPorcln}(x)), \text{PorcQual}), s)) \quad (44)
\]

From (43), (44) and theorem 8a (24), we have,

\[
\text{holds}(\text{trust}_p(F, J, \text{makeHGPorcln}(x)), \text{in}_\text{topic}(\text{perform}(J, \text{makeHGPorcln}(x)), \text{PorcQual}), s)) \quad (45)
\]

This formula gives the answer that F can trust in J’s performance that J’s porcelain products have high quality. Because of this trust, F presents to its customers J’s products.

**Case 2:** we can have the following formal representation.

F trusts in the product quality of the enterprises in compliance with ISO-9000.

\[
\text{holds}(\text{trust}_p(F, \text{ISO9000Ents}, \text{perform}(\text{ISO9000Ents}, \text{makeHGPorcln}(x)), \text{in}_\text{topic}(\text{perform}(\text{ISO9000Ents}, \text{makeHGPorcln}(x)), \text{PorcQual}), s)) \quad (46)
\]

J has ISO-9000 certification, that is, J is an enterprise in compliance with ISO-9000.

\[
\text{holds}(\text{memberOf}(J, \text{ISO9000Ents}), s) \quad (47)
\]

From (46), (47) and axiom 4 (18), we have (45). In this way, F also trusts in J’s product quality.

This example shows that the proposed trust ontology is able to be used in trust judgments using social networks. This ontology can be used as a kernel logic part in specific trust models for particular domains. In practice, a specific trust judgment model in a particular domain can be built by incorporating this trust ontology and domain-dependent knowledge.

9. **CONCLUSION AND DISCUSSION**

Making trust judgments by using social networks is a promising approach for addressing the trust problem on the Web. The necessary condition for trust propagation in social networks is that trust is transitive. This paper created a logical theory of trust in the form of ontology that gives a formal and explicit specification for the semantics of trust. From this formal semantics, two types of trust – trust in belief and trust in performance were identified, the transitivity of trust in belief was formally proved, and three conditions for trust propagation were derived. These results answered the questions of ‘is trust transitive, what types of trust are transitive and why’, and provide theoretical evidence for trust propagation in social networks.

Although many formal trust models have been proposed (refer to section 2), the transitivity of trust has not been studied in formal manner. Although [1] and [18] argued that ‘recommendation trust’ is transitive, but neither gives a formal description. In addition, ‘recommendation trust’ is a specific case of trust in belief; the latter is more general.

As illustrated in the application example of trust judgments in web services based e-business, the proposed trust ontology can be used as a logical tool to support trust judgments using social networks on the Web.

The proposed logic model of trust is general. There may be many different web implementations, but the logic behind these implementations is the same.

In this paper, we focused on the logical representation of the semantics of trust and the transitivity of trust, which is mainly targeted to support social networks based trust. This formal semantics also supports modeling inter-individual trust and system trust.

The model proposed in this paper is a certainty model. Although this model can be used in real trust judgments, it
is important to extend the model to uncertainty model, because uncertainty widely exists in trust problems, especially in the cyberspace. In fact, we are in the process to extend the proposed certainty model to uncertainty model.

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11. REFERENCES